[Math]レベルチェックテスト問題

1. Consider the sequence $2, 6, 10, 14, 18, ...$
   (a) Show that the sequence is arithmetic.
   (b) Find a formula for the general term u_n.
   (c) Find the 15th term of the sequence.

Answer:

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2. A quadratic function $f(x)$ has the vertex $(-1, 9)$ and the x-intercepts at 2 and -4.

(a) The function $f(x)$ can be written in the form of $f(x) = (x-h)^2 + k$.
Write down the value of h and k.

(b) The function can also be written in the form $f(x) = (x-p)(x-q)$.
Write down the value of p and q.

(c) Find the y-intercept.

Answer:

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3. In triangle ABC, AB=7cm, BC=5cm and CA=8cm.

(a) Find the measure of angle BCA.

(b) Find the exact area of triangle ABC.

Answer:

4. By using a calculator find

   (a) angle *L* in triangle KLM given that angle *K* measures 56°, LM= 16.8cm, and KM= 13.5 cm

   (b) angle BAC, given a=14.6cm, b= 17.4cm, and angle ABC=65°

Answer:

5. Given $f(x) = \sin x$ and $g(x) = 6^x$, find:

(a) $(f \circ g)(x)$

(b) $(g \circ f)(x)$

Answer:

[Math]レベルチェックテスト解答

1. (a) Show that the common difference is 4

$6 - 2 = 4$

$10 - 6 = 4$

$14 - 10 = 4$

$18 - 14 = 4$

(b) Using the first term u_1 and common difference d

$u_n = u_1 + (n-1)d$

$u_n = 2 + (n-1)4 = 4n - 2$

(c) Using the formula from (b)

If $n = 15$, $u_{15} = 4 \cdot 15 - 2 = 58$

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2. (a) As the vertex is $(-1, 9)$,
$h = 1, k = 9$

(b) As the x-intercepts are 2 and -4,
$p = 2, q = -4$ (or $p = -4, q = 2$)

(c) **Method 1**
Substitute $x = 0$ into $f(x) = (x-1)^2 - 9$.
$f(0) = (-1)^2 - 9 = -8$
$y = -8$

Method 2
Substitute $x = 0$ into $f(x) = (x-2)(x+4)$
$f(0) = (0-2)(0+4) = -8$
$y = -8$

3. (a) By the cosine rule:

$$\cos C = \frac{(5^2 + 8^2 - 7^2)}{(2 \times 5 \times 8)}$$

$$\therefore C = \cos^{-1}\left(\frac{5^2 + 8^2 - 7^2}{2 \times 5 \times 8}\right)$$

$$\therefore C = \cos^{-1}\left(\frac{5^2 + 8^2 - 7^2}{2 \times 5 \times 8}\right)$$

$$\therefore C \approx 60°$$

So, the angle BCA measures $60°$

(b) The area of triangle ABC is

$$\frac{1}{2} \times 8 \times 5 \times \sin 60° = 20 \times \frac{\sqrt{3}}{2}$$

$$\therefore \text{The area of triangle ABC} = 10\sqrt{3}\text{cm}^2$$

4. (a) Using the sine rule:

$$\frac{\sin L}{13.5} = \frac{\sin 56°}{16.8}$$

$$\therefore \sin L = \frac{13.5 \times \sin 56°}{16.8}$$

$$\therefore L = \sin^{-1}\left(\frac{13.5 \times \sin 56°}{16.8}\right)$$

$$\therefore L \approx 41.8° \; or \; 180° - 41.8°$$

$$\therefore L \approx 41.6° \; or \; 138.2°$$

We reject $L \approx 138.2°$, since $138.2° + 56° > 180°$ which is impossible in a triangle.

$\therefore L \approx 41.6°$, a unique solution in this case.

(b) Using the sine rule:

$$\frac{\sin BAC}{14.6} = \frac{\sin 65°}{17.4}$$

$$\therefore \sin BAC = \frac{14.6 \times \sin 65°}{17.4}$$

$$\therefore BAC = \sin^{-1}\left(\frac{14.6 \times \sin 65°}{17.4}\right)$$

$$\therefore BAC \approx 49.5°$$

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5. (a) $(f \circ g)(x) = f(6^x) = \sin(6^x)$

(b) $(g \circ f)(x) = g(\sin x) = 6^{\sin x}$